



The dipole form of the quark part of the BFKL kernel ☆

V.S. Fadin ^{a,b}, R. Fiore ^c, A. Papa ^{c,*}^a Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia^b Novosibirsk State University, 630090 Novosibirsk, Russia^c Dipartimento di Fisica, Università della Calabria, Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza, Arcavacata di Rende, I-87036 Cosenza, Italy

Received 24 January 2007; accepted 5 February 2007

Available online 16 February 2007

Editor: L. Alvarez-Gaumé

Abstract

The dipole form of the “Abelian” part of the massless quark contribution to the BFKL kernel is found in the coordinate representation by direct transfer from the momentum representation where the contribution was calculated before. It coincides with the corresponding part of the quark contribution to the dipole kernel calculated recently by Balitsky and is conformal invariant.

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1. Introduction

The BFKL approach [1] for the theoretical description of high-energy processes is well developed now in the next-to-leading approximation (NLA). In particular, the kernel of the BFKL equation is known in the next-to-leading order (NLO) not only for the forward scattering [2], i.e., for $t = 0$ and the color singlet in the t -channel, but also for arbitrary fixed (not growing with energy) momentum transfer t and any possible color state in the t -channel [3–5]. The color singlet representation is certainly the most important for phenomenological applications, since it is relevant for physical processes, where the colliding particles are colorless, although from the theoretical point of view the color octet representation seems even more important because of the gluon Reggeization. The Reggeized gluon occurs to be the primary Reggeon in the high-energy QCD, which can be reformulated in terms of a gauge-invariant effective field theory for the Reggeized gluon interactions [6].

All the results mentioned above for the BFKL kernel have been obtained so far in the momentum representation. However, considering the singlet BFKL kernel in the coordinate representation in the transverse space may have several theoretical benefits.

Indeed, the famous property of conformal invariance of the BFKL equation in the leading approximation [7] is connected with the coordinate representation. This property is extremely important for finding solutions of the equation and it is therefore very useful to investigate the conformal properties of the BFKL kernel in the NLO. Here, an evident source of the breaking of conformal invariance is the renormalization, but it is important to know if this is the only source of violation. If so, one can rely on the conformal invariance of the NLO BFKL in supersymmetric extensions of QCD.

Moreover, it is in the coordinate representation that the color dipole approach to high-energy scattering [8], very popular now, is formulated. Beside giving a clear physical picture of the high-energy processes, the color dipole approach can be naturally extended from the regime of low parton densities to the saturation regime [9], where the evolution equations of parton densities with energy become nonlinear. In general, there is an infinite hierarchy of coupled equations [10,11]. In the simplest case of a large nucleus as target, this set of equations is reduced to the BK (Balitsky–Kovchegov) equation [10].

☆ Work supported in part by the Russian Fund of Basic Researches and in part by Ministero Italiano dell’Istruzione, dell’Università e della Ricerca.

* Corresponding author.

E-mail addresses: fadin@inp.nsk.su (V.S. Fadin), fiore@cs.infn.it (R. Fiore), papa@cs.infn.it (A. Papa).

The theoretical description of small- x processes could take advantage from a clear understanding of the relation between the BFKL and the color dipole approaches. It is affirmed [8,10], that in the linear regime the color dipole gives the same results as the BFKL approach for the color singlet channel. However, a full insight into this relation and its extension to the NLO requires to bridge the gap between the different formulations adopted by the two approaches: the color dipole approach uses the coordinate representation in the transverse space, whereas the BFKL approach was originally formulated in the momentum one.

Before the advent of the dipole approach, the leading order color singlet BFKL kernel has been investigated in the coordinate representation in detail in Ref. [7]. More recently, the relation between BFKL and color dipole was analyzed in the leading order in Ref. [12]. The extension of the analysis to the NLO has started in the last few months. In Ref. [13] the quark contribution in the color dipole approach at large number of colors N_c has been transferred from the coordinate to the momentum representation and it has been verified that the resulting contribution to the NLO Pomeron intercept agrees with the well-known result of Ref. [2]. In our previous paper [14] we have transformed the “non-Abelian” (leading in N_c) part of the quark contribution to the non-forward BFKL kernel from the momentum representation where it was calculated before [3] to the coordinate one and have found that its dipole form is in accord with the result obtained recently in Ref. [15] by direct calculation of the quark contribution to the dipole kernel in the coordinate representation. In this Letter we consider the “Abelian” (suppressed by $1/N_c^2$) part of the quark contribution to the non-forward BFKL kernel [3].

The Letter is organized as follows: in Section 2 we give the basic definitions, fix our notations and recall the main results of our previous paper [14]; in Section 3 we describe the procedure to transfer the “Abelian” part of the NLO quark contribution from the momentum to the coordinate representation and present our result; in Section 4 we draw our conclusions.

2. Basic definitions and notation

We use the same notation as in Ref. [14]: \vec{q}'_i and \vec{q}_i , $i = 1, 2$, represent the transverse momenta of Reggeons in initial and final t -channel states, while \vec{r}'_i and \vec{r}_i are the corresponding conjugate coordinates. The state normalization is

$$\langle \vec{q} | \vec{q}' \rangle = \delta(\vec{q} - \vec{q}'), \quad \langle \vec{r} | \vec{r}' \rangle = \delta(\vec{r} - \vec{r}'), \quad (1)$$

so that

$$\langle \vec{r} | \vec{q} \rangle = \frac{e^{i\vec{q}\vec{r}}}{(2\pi)^{1+\epsilon}}, \quad (2)$$

where $\epsilon = (D - 4)/2$; $D - 2$ is the dimension of the transverse space and is taken different from 2 for the regularization of divergences. Note that in our papers previous to Ref. [14] we denoted the initial (final) momenta as \vec{q}_1 and $-\vec{q}'_1$ (\vec{q}_2 and $-\vec{q}'_2$) and used the normalization $\langle \vec{q} | \vec{q}' \rangle = \vec{q}^2 \delta(\vec{q} - \vec{q}')$. We will use also the notation $\vec{q} = \vec{q}_1 + \vec{q}_2$, $\vec{q}' = \vec{q}'_1 + \vec{q}'_2$; $\vec{k} = \vec{q}_1 - \vec{q}'_1 = \vec{q}'_2 - \vec{q}_2$. The BFKL kernel in the operator form is written as

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r, \quad (3)$$

where

$$\langle \vec{q}_i | \hat{\omega}_i | \vec{q}'_i \rangle = \delta(\vec{q}_i - \vec{q}'_i) \omega(-\vec{q}_i^2), \quad (4)$$

with $\omega(t)$ the gluon Regge trajectory, and $\hat{\mathcal{K}}_r$ represents real particle production in Reggeon collisions. The s -channel discontinuities of scattering amplitudes for the processes $A + B \rightarrow A' + B'$ have the form

$$-4i(2\pi)^{D-2} \delta(\vec{q}_A - \vec{q}_B) \text{disc}_s \mathcal{A}_{AB}^{A'B'} = \langle A' \bar{A} | e^{Y\hat{\mathcal{K}}} \frac{1}{q_1^2 q_2^2} | \bar{B}' B \rangle. \quad (5)$$

In this equation $Y = \ln(s/s_0)$, s_0 is an appropriate energy scale, $q_A = p_{A'} - p_A$, $q_B = p_B - p_{B'}$, and

$$\langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}} | \vec{q}'_1, \vec{q}'_2 \rangle = \delta(\vec{q} - \vec{q}') \frac{1}{q_1^2 q_2^2} \mathcal{K}(\vec{q}_1, \vec{q}'_1; \vec{q}), \quad (6)$$

$$\langle \vec{q}_1, \vec{q}_2 | \bar{B}' B \rangle = 4p_B^- \delta(\vec{q}_B - \vec{q}_1 - \vec{q}_2) \Phi_{B'B}(\vec{q}_1, \vec{q}_2), \quad (7)$$

$$\langle A' \bar{A} | \vec{q}_1, \vec{q}_2 \rangle = 4p_A^+ \delta(\vec{q}_A - \vec{q}_1 - \vec{q}_2) \Phi_{A'A}(\vec{q}_1, \vec{q}_2), \quad (8)$$

where $p^\pm = (p_0 \pm p_z)/\sqrt{2}$; the kernel $\mathcal{K}(\vec{q}_1, \vec{q}'_1; \vec{q})$ and the impact factors Φ are expressed through the Reggeon vertices according to Ref. [16]. Note that the appearance of the factors $(\hat{q}_1^2 \hat{q}_2^2)^{-1}$ in (5) and $(\vec{q}_1^2 \vec{q}_2^2)^{-1}$ in (6) cannot be explained by a change of the normalization (1). We have used a freedom in the definition of the kernel. In fact, one can change the form of the kernel performing the transformation

$$\hat{\mathcal{K}} \rightarrow \hat{\mathcal{O}}^{-1} \hat{\mathcal{K}} \hat{\mathcal{O}}, \quad \langle A' \bar{A} | \rightarrow \langle A' \bar{A} | \hat{\mathcal{O}}, \quad \frac{1}{q_1^2 q_2^2} | \bar{B}' B \rangle \rightarrow \hat{\mathcal{O}}^{-1} \frac{1}{q_1^2 q_2^2} | \bar{B}' B \rangle, \quad (9)$$

which does not change the discontinuity (5). In (9) $\hat{\mathcal{O}}$ is an arbitrary nonsingular operator. The kernel $\hat{\mathcal{K}}$ in (6) is related with the one defined in Ref. [16] by such transformation with $\hat{\mathcal{O}} = (\hat{q}_1^2 \hat{q}_2^2)^{1/2}$. The reason for this choice is that in the leading order the kernel which is conformal invariant and is simply related to the dipole kernel is not the kernel defined in Ref. [16], but just the kernel $\hat{\mathcal{K}}$ in (6) [7,12]. Note that after the choice of $\hat{\mathcal{O}}$ in the leading order, transformations with $\hat{\mathcal{O}} = 1 - \hat{\mathcal{O}}$, where $\hat{\mathcal{O}} \sim g^2$, are still possible. At the NLO after such transformation we get

$$\hat{\mathcal{K}} \rightarrow \hat{\mathcal{K}} - [\hat{\mathcal{K}}^{(B)}, \hat{\mathcal{O}}], \quad (10)$$

where $\hat{\mathcal{K}}^{(B)}$ is the leading order kernel.

In Ref. [14] we performed the transformation to the coordinate representation of the “non-Abelian” part of the quark contribution to the kernel $\langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}} | \vec{q}'_1, \vec{q}'_2 \rangle$ (6) which was found in [3]. The transformation was performed in the most general way: at arbitrary D and for the case of arbitrary impact factors. In the case of scattering of colorless objects besides the freedom of definition of the kernel discussed above there is an additional freedom related with the “gauge invariance” (vanishing at zero Reggeized gluon momenta) of the impact factors [7,12]. In this case the kernel $\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}} | \vec{r}'_1 \vec{r}'_2 \rangle$ can be written in the dipole form (see below). Because of the possibility of transformations (10) the dipole form is not unique. We transferred to the dipole form the “non-Abelian” part of the kernel [3] and found that, after the transformation (10) with a suitable operator $\hat{\mathcal{O}}$, it agrees with the result obtained recently in Ref. [15] by direct calculation of the quark contribution to the dipole kernel in the coordinate representation. Here we consider also the “Abelian” part of the kernel and find its dipole form.

3. The “Abelian” part of the quark contribution

The “Abelian” part of the quark contribution to the NLO kernel is suppressed by the factor $1/N_c^2$ in comparison with the “non-Abelian” one [3]. The gluon trajectory has only leading N_c contribution. Therefore, the “Abelian” contribution comes only from the real part of the kernel. This part contains neither ultraviolet nor infrared singularities and therefore it does not require regularization nor renormalization. Therefore we can use from the beginning physical space-time dimension $D = 4$ and the renormalized coupling constant $\alpha_s(\mu)$. According to Eqs. (37), (38), (45) and (49) of Ref. [3] the “Abelian” contribution in the momentum representation can be written as

$$\langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}}^a | \vec{q}'_1, \vec{q}'_2 \rangle = \delta(\vec{q} - \vec{q}') \frac{\alpha_s^2(\mu) n_f}{(2\pi)^2 N_c} \frac{-2}{\vec{q}_1^2 \vec{q}_2^2} \int_0^1 dx \int \frac{d^2 k_1}{(2\pi)^2} F(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2), \quad (11)$$

with

$$\begin{aligned} F(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2) = & x(1-x) \left(\frac{2(\vec{q}_1 \vec{k}_1) - \vec{q}_1^2}{\sigma_{11}} + \frac{2(\vec{q}_1 \vec{k}_2) - \vec{q}_1^2}{\sigma_{21}} \right) \left(\frac{2(\vec{q}_2 \vec{k}_1) + \vec{q}_2^2}{\sigma_{12}} + \frac{2(\vec{q}_2 \vec{k}_2) + \vec{q}_2^2}{\sigma_{22}} \right) \\ & + \frac{x \vec{q}_2^2 (2(\vec{q}_1 \vec{k}_1) - \vec{q}_1^2)}{2\sigma_{11}} \left(\frac{1}{\sigma_{22}} - \frac{1}{\sigma_{12}} \right) + \frac{x \vec{q}_2^2 (2(\vec{q}_2 \vec{k}_1) + \vec{q}_2^2)}{2\sigma_{12}} \left(\frac{1}{\sigma_{11}} - \frac{1}{\sigma_{21}} \right) \\ & + \frac{1}{\sigma_{11} \sigma_{22}} \left(-2(\vec{q}_1 \vec{k}_1)(\vec{q}_2 \vec{q}'_2) - 2(\vec{q}_2 \vec{k}_1)(\vec{q}_1 \vec{q}'_1) + (\vec{q}_2^2 - \vec{q}_1^2)(\vec{k}_1 \vec{k}_2) + \vec{q}_1^2 \vec{q}_2'^2 - \frac{\vec{k}_2^2 \vec{q}_2^2}{2} \right), \end{aligned} \quad (12)$$

where $\vec{k}_1 + \vec{k}_2 = \vec{k} = \vec{q}_1 - \vec{q}'_1 = \vec{q}'_2 - \vec{q}_2$,

$$\begin{aligned} \sigma_{11} &= (\vec{k}_1 - x \vec{q}_1)^2 + x(1-x) \vec{q}_1^2, & \sigma_{21} &= (\vec{k}_2 - (1-x) \vec{q}_1)^2 + x(1-x) \vec{q}_1^2, \\ \sigma_{12} &= (\vec{k}_1 + x \vec{q}_2)^2 + x(1-x) \vec{q}_2^2, & \sigma_{22} &= (\vec{k}_2 + (1-x) \vec{q}_2)^2 + x(1-x) \vec{q}_2^2. \end{aligned} \quad (13)$$

It is easy to see that $F(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2)$ vanishes when any of the \vec{q}_i 's or \vec{q}'_i 's tends to zero. The vanishing of $\langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}}^a | \vec{q}'_1, \vec{q}'_2 \rangle$ at $\vec{q}'_i = 0$ together with the “gauge invariance” property of the impact factors of colorless “projectiles” $\Phi_{A'A}(0, \vec{q}) = \Phi_{A'A}(\vec{q}, 0)$ permits to omit in $\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}^a | \vec{r}'_1 \vec{r}'_2 \rangle$ terms which do not depend either on \vec{r}_1 or on \vec{r}_2 . Moreover, it permits to change in (5) the “target” impact factors so that they acquire the “dipole” property

$$\langle \vec{r}, \vec{r} | (\hat{q}_1^2 \hat{q}_2^2)^{-1} | \vec{B}' B \rangle_d = 0 \quad (14)$$

(see Ref. [14] for details). Thereafter, terms proportional to $\delta(\vec{r}'_1 - \vec{r}'_2)$ in $\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}^a | \vec{r}'_1 \vec{r}'_2 \rangle$ can also be omitted, assuming that the remaining part $\hat{\mathcal{K}}_d^a$ conserves the “dipole” property. We call this part the dipole form of the BFKL kernel.

With our normalizations, $\hat{\mathcal{K}}^a$ in the coordinate representation is given by

$$\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}^a | \vec{r}'_1 \vec{r}'_2 \rangle = -2 \frac{\alpha_s^2(\mu) n_f}{(2\pi)^4 N_c} \int_0^1 dx \int \frac{d^2 q_1}{2\pi} \frac{d^2 q_2}{2\pi} \frac{d^2 k_1}{2\pi} \frac{d^2 k_2}{2\pi} \frac{F(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2)}{\vec{q}_1^2 \vec{q}_2^2} e^{i[\vec{q}_1(\vec{r}_1 - \vec{r}'_1) + \vec{q}_2(\vec{r}_2 - \vec{r}'_2) + \vec{k}(\vec{r}'_1 - \vec{r}'_2)]}. \quad (15)$$

If we restrict ourselves to the dipole form, then, omitting the terms with $\delta(\vec{r}'_1 - \vec{r}'_2)$, we can change the integrand as follows:

$$F(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2) \rightarrow \frac{1}{\sigma_{11}\sigma_{22}} \left[2x(1-x)(2\vec{q}_1\vec{k}_1 - \vec{q}_1^2)(2\vec{q}_2\vec{k}_2 + \vec{q}_2^2) + \frac{\vec{q}_2^2}{2}(x(2\vec{q}_1\vec{k}_1 - \vec{q}_1^2) - (1-x)(2\vec{q}_2\vec{k}_2 + \vec{q}_2^2)) \right. \\ \left. - 2(\vec{q}_1\vec{k}_1)(\vec{q}_2\vec{q}'_2) - 2(\vec{q}_2\vec{k}_1)(\vec{q}_1\vec{q}'_1) + (\vec{q}_2^2 - \vec{q}_1^2)(\vec{k}_1\vec{k}) + \vec{q}_1^2\vec{q}'_2{}^2 - \frac{\vec{k}^2\vec{q}^2}{2} \right]. \quad (16)$$

Note that the “Abelian” part of the quark contribution is given by the “box” and “cross-box” diagrams; the “box” diagrams give only terms proportional to $\delta(\vec{r}'_1 - \vec{r}'_2)$, whereas all contribution of “cross-box” diagrams is retained in (16). It is well known that in the momentum representation this contribution is the most complicated one. On the contrary, in the coordinate representation it is very simple. The integrals can be calculated quite easily. The integrations over \vec{k}_1, \vec{q}_1 and \vec{k}_2, \vec{q}_2 can be done independently. It is convenient to introduce the new variables \vec{l}_1 and \vec{l}_2 ,

$$\vec{k}_1 = \vec{l}_1 + x\vec{q}_1, \quad \vec{k}_2 = \vec{l}_2 - (1-x)\vec{q}_2, \quad \sigma_{11} = \vec{l}_1^2 + x(1-x)\vec{q}_1^2, \quad \sigma_{22} = \vec{l}_2^2 + x(1-x)\vec{q}_2^2. \quad (17)$$

Omitting the terms with $\delta(\vec{r}'_1 - \vec{r}'_2)$, we obtain in terms of these variables

$$F(\vec{q}_1, \vec{q}_2; \vec{k}_1, \vec{k}_2) \rightarrow \frac{1}{\sigma_{11}\sigma_{22}} \left[-8x^2(1-x)^2\vec{q}_1^2\vec{q}_2^2 - 4x(1-x)(1-2x)(\vec{q}_1^2(\vec{q}_2\vec{l}_2) + \vec{q}_2^2(\vec{q}_1\vec{l}_1)) \right. \\ \left. + 2(4x(1-x) - 1)(\vec{q}_1\vec{l}_1)(\vec{q}_2\vec{l}_2) + 2(\vec{q}_2\vec{l}_1)(\vec{q}_1\vec{l}_2) - 2(\vec{q}_1\vec{q}_2)(\vec{l}_1\vec{l}_2) \right]. \quad (18)$$

The only integrals we need are

$$\int \frac{d^2q}{2\pi} \frac{d^2l}{2\pi} \frac{e^{i\vec{q}\vec{r} + i\vec{l}\vec{\rho}}}{\vec{l}^2 + x(1-x)\vec{q}^2} = \frac{1}{\vec{r}^2 + x(1-x)\vec{\rho}^2}, \\ \int \frac{d^2q}{2\pi} \frac{d^2l}{2\pi} \frac{q_i l_j}{\vec{q}^2} \frac{e^{i\vec{q}\vec{r} + i\vec{l}\vec{\rho}}}{\vec{l}^2 + x(1-x)\vec{q}^2} = \frac{-r_i \rho_j}{\vec{\rho}^2(\vec{r}^2 + x(1-x)\vec{\rho}^2)}. \quad (19)$$

We obtain

$$\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}^a | \vec{r}'_1 \vec{r}'_2 \rangle = \frac{\alpha_s^2(\mu) n_f}{4\pi^4 N_c} \int_0^1 \frac{dx}{d_1 d_2 (\vec{r}'_1 - \vec{r}'_2)^4} \left[((\vec{r}_1 - \vec{r}'_2)^2 - (\vec{r}_1 - \vec{r}'_1)^2)((\vec{r}_2 - \vec{r}'_1)^2 - (\vec{r}_2 - \vec{r}'_2)^2)x(1-x) \right. \\ \left. + (\vec{r}'_1 - \vec{r}'_2)^2((\vec{r}'_1 - \vec{r}'_2)(x(\vec{r}_2 - \vec{r}'_2) - (1-x)(\vec{r}_1 - \vec{r}'_1)) + (\vec{r}_1 - \vec{r}'_1)(\vec{r}_2 - \vec{r}'_2)) \right] \\ + \text{terms with } \delta(\vec{r}'_1 - \vec{r}'_2). \quad (20)$$

Here $d_1 = x(\vec{r}_1 - \vec{r}'_2)^2 + (1-x)(\vec{r}_1 - \vec{r}'_1)^2$ and $d_2 = x(\vec{r}_2 - \vec{r}'_2)^2 + (1-x)(\vec{r}_2 - \vec{r}'_1)^2$. The subsequent integration over x is elementary. We need only the integrals

$$\int_0^1 \frac{dx}{d_1 d_2} = \frac{L_+}{d}, \\ \int_0^1 \frac{x dx}{d_1 d_2} = \frac{1}{d} \left(\frac{c_2}{b_2} l_2 - \frac{a_1}{b_1} l_1 \right), \\ \int_0^1 \frac{(1-x) dx}{d_1 d_2} = \frac{1}{d} \left(\frac{c_1}{b_1} l_1 - \frac{a_2}{b_2} l_2 \right), \\ \int_0^1 \frac{x(1-x) dx}{d_1 d_2} = \frac{1}{b_1 b_2} \left[1 - \frac{1}{d} \left(\frac{a_1 c_1 b_2}{b_1} l_1 + \frac{a_2 c_2 b_1}{b_2} l_2 \right) \right], \quad (21)$$

where

$$a_1 = (\vec{r}_1 - \vec{r}'_1)^2, \quad a_2 = (\vec{r}_2 - \vec{r}'_2)^2, \quad c_1 = (\vec{r}_1 - \vec{r}'_2)^2, \quad c_2 = (\vec{r}_2 - \vec{r}'_1)^2, \\ b_i = c_i - a_i, \quad d = c_1 c_2 - a_1 a_2, \quad l_i = \ln \left(\frac{c_i}{a_i} \right), \quad L_+ = l_1 + l_2. \quad (22)$$

Calculating the integrals with the help of these formulas, we obtain, up to terms with $\delta(\vec{r}'_1 - \vec{r}'_2)$,

$$\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}^a | \vec{r}'_1 \vec{r}'_2 \rangle = \frac{\alpha_s^2(\mu) n_f}{4\pi^4 N_c} \frac{1}{(\vec{r}'_1 - \vec{r}'_2)^4} \left[\left(\frac{c_1 c_2 + a_1 a_2 - (\vec{r}_1 - \vec{r}_2)^2 (\vec{r}'_1 - \vec{r}'_2)^2}{2d} L_+ - 1 \right) - \left(\frac{c_1 + a_1 - (\vec{r}'_1 - \vec{r}'_2)^2}{2b_1} l_1 - 1 \right) - \left(\frac{c_2 + a_2 - (\vec{r}'_1 - \vec{r}'_2)^2}{2b_2} l_2 - 1 \right) \right]. \quad (23)$$

The terms in the last line do not depend either on \vec{r}_1 or \vec{r}_2 and can therefore be omitted due to the gauge invariance of impact factors. The remaining part turns into zero at $\vec{r}_1 = \vec{r}_2$ (conserves the “dipole” property), so that it represents the dipole form of the “Abelian” part of the quark contribution to the BFKL kernel. It coincides with the corresponding part of the quark contribution to the dipole kernel calculated recently in Ref. [15] and is evidently conformal invariant.

4. Conclusion

The coordinate representation of the BFKL kernel is extremely interesting, because it gives the possibility to understand its conformal properties and the relation between the BFKL and the color dipole approaches. Joining the results of Ref. [14] and of the present Letter, we have obtained the quark contribution to the BFKL kernel in the next-to-leading order in the coordinate representation in the transverse space, by transformation from the momentum representation, in which the BFKL kernel was known before. For scattering of colorless objects, due to the “gauge invariance” of the impact factors, the kernel in the coordinate representation can be written in the dipole form. We have found that the dipole form of the quark contribution agrees with the result obtained recently in Ref. [15] by direct calculation of the quark contribution to the dipole kernel in the coordinate representation. The agreement is reached after some transformations of the original BFKL kernel which do not change the scattering amplitudes, being supplemented by the corresponding transformation of the impact factors of colliding particles.

As for the “Abelian” part of the kernel, which is the subject of the present Letter, it turns out that its dipole form is quite simple as compared with the very complicated form in the momentum representation. Actually, up to a coefficient, this part coincides [3] with the kernel for the QED Pomeron [17], so that its complexity in the momentum representation has been known for a long time. Moreover, the dipole form of the “Abelian” part of the kernel is conformal invariant. It could be especially interesting for the QED Pomeron. However, one has to remember that in QED the use of the dipole form is limited to scattering of neutral objects, as well as that the conformal invariance is broken by masses.

Acknowledgement

A.P. thanks A. Sabio Vera for several useful discussions.

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